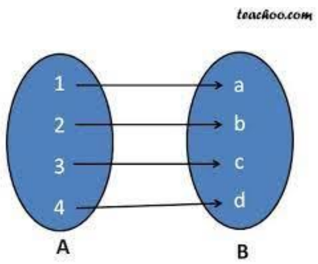
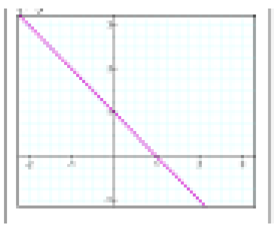
Functions

A. DEFINITION

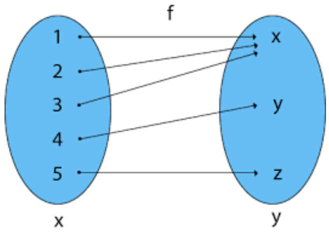
- Mapping of an x-value to a y-value, for any value of x there is only one possible answer
- Domain: x values
- Range: y values
- Function can be written in any of the following ways:
  1. $y = x^2$
  2. $f(x) = x^2$
  3. $f : x \to x^2$
- Types of function mapping

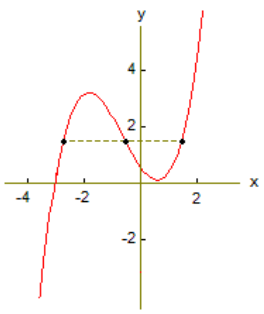
a. One-one





b. Many-one

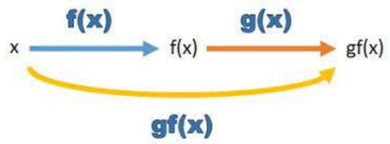




B. COMPOSITE FUNCTIONS

- A function with another function as an input

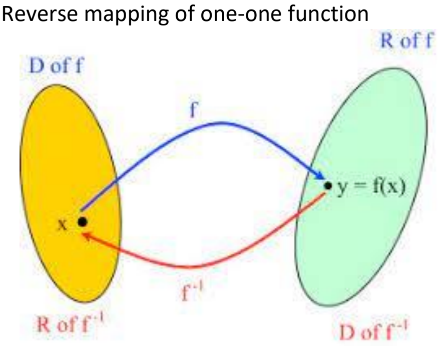
Notation



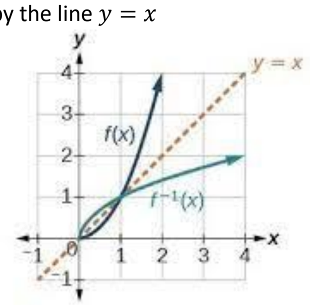
$$fg(x) = f(g(x))$$

C. INVERSE FUNCTIONS

- Reverse mapping of one-one function



- Notation: $f^{-1}(x)$
- Graph of inverse function:
  The graph of $y = f(x)$ and $f^{-1}(x)$ is symmetrical by the line $y = x$



- Property of inverse function:

$$ff^{-1}(x) = f^{-1}f(x) = x$$

- Finding the algebraic form of the inverse function
  1. Write $f(x)$ as y
  2. Make x the subject
  3. Swap every single x with y. By now, y should be the subject
  4. Replace y with $f^{-1}(x)$

- Example:
  Find $f^{-1}(x)$ when $f(x) = 2x + 1, x \in \mathbb{R}$
  Solution:

$$y = 2x + 1$$
$$2x = y - 1$$
$$x = \frac{y - 1}{2}$$
$$y = \frac{x - 1}{2}$$

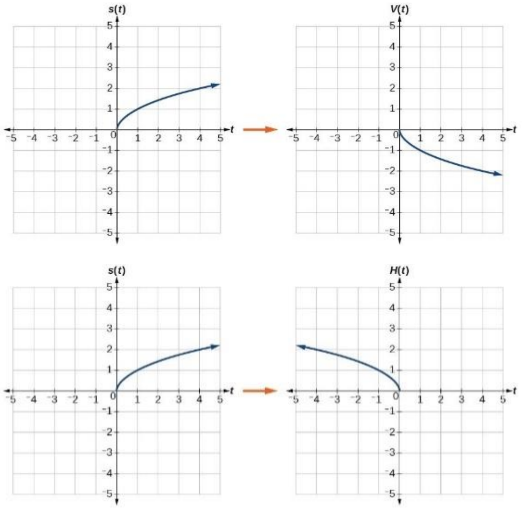
$$f^{-1}(x) = \frac{x-1}{2}$$

Using transformations to sketch curves

1. Transformation of the graphs of the function $y = f(x)$

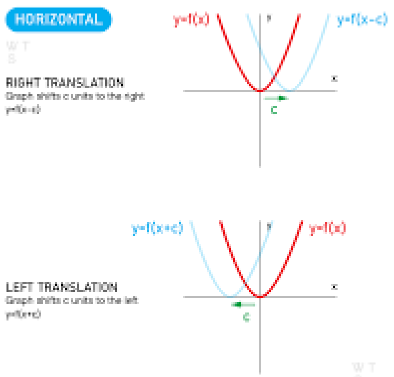
Function	Transformation
$f(x-t)+s$	Translation $\binom{t}{s}$
$-f(x)$	Reflection in x-axis
$f(-x)$	Reflection in y-axis
$af(x)$	One-way stretch, parallel to y-axis, scale factor a
$f(ax)$	One-way stretch, parallel to x-axis, scale factor $\frac{1}{a}$

2. Translation

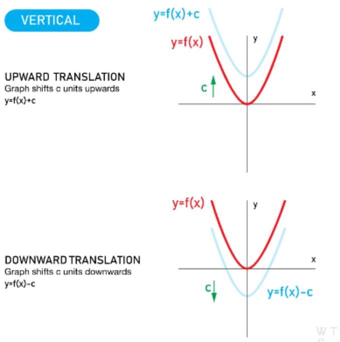


3. Reflection

a. About the x-axis

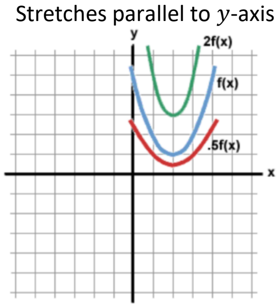


b. About the y-axis



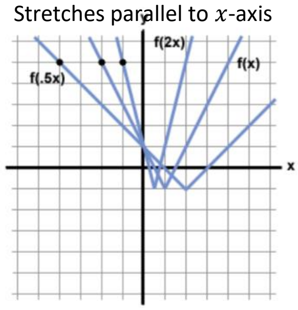
c. Stretches

1) Stretches parallel to y-axis



Graphs of $f(x)$, $2f(x)$, and $\frac{1}{2}f(x)$

2) Stretches parallel to x-axis



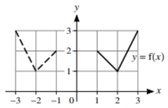
Graphs of $f(x)$, $f(\frac{1}{2}x)$, and $f(2x)$

D. EXERCISE

1. [9709_S20_qp_13_003]

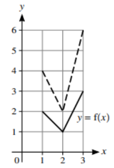
In each of parts (a), (b) and (c), the graph shown with solid lines has equation $y = f(x)$. The graph shown with broken lines is a transformation of $y = f(x)$. 3

(a)



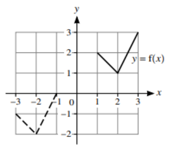
State, in terms of f, the equation of the graph shown with broken lines. [1]

(b)



State, in terms of f, the equation of the graph shown with broken lines. [1]

(c)



State, in terms of f, the equation of the graph shown with broken lines. [2]

Solution

(a) Reflection in y-axis, $y = f(-x)$

(b) Pay attention to the point $(2,1)$ on the $f(x)$ graph, in the new graph that point is stretched to the point $(2,2)$. We can conclude that it is stretched by scale factor 2 parallel to the y-axis. So, the new function is $y = 2f(x)$

(c) Pay attention to the point $(2,1)$ on the $f(x)$ graph, in the new graph that point is translated to the point $(-2,-2)$ by the matrix

$$\begin{pmatrix} -2-2 \\ -2-1 \end{pmatrix} = \begin{pmatrix} -4 \\ -3 \end{pmatrix}$$

So, the new function is

$$y = f(x+4) - 3$$

2. [9709_S10_qp_12_003]

The function f and g are defined for $x \in \mathbb{R}$ by

$$f : x \to 4x - 2x^2$$
$$g : x \to 5x + 3$$

(i) Find the range f

(ii) Find the value of the constant k for which the equation $gf(x) = k$ has equal roots

Solution

(i) $f(x) = -2(x-1)^2 + 2$

Turning point at $x = 1$

Since no square is negative, $f(x)$ must be ≤ 2

So, the range of f is $f(x) \leq 2$

(ii)
$$gf(x) = g(f(x))$$
$$= 5(4x - 2x^2) + 3$$
$$= -10x^2 + 20x + 3 = k$$
$$-10x^2 + 20x + 3 - k = 0$$

Since it has equal roots, the discriminant must be = 0,

$$D = 20^2 + 4(10)(3-k) = 0$$
$$40k = 520$$
$$k = 13$$