Forces, Density, and Pressure

PHYSICS

A. TURNING EFFECTS OF FORCES

- **Centre of Gravity** is the point at which the weight of an object may be taken as acting on a single point
- **Moment of Force about a point** is the product of **the force (Newton)** and the **perpendicular distance (meter)** from the point to the **line of action of the force**

$$\tau = Fd$$

- A **couple** is a **pair of forces**, **equal in magnitude** and **in opposite direction**, which produces **rotation only**
- **The torque of a couple** is the product of **one of the forces** and the **perpendicular distance between** the forces

B. EQUILIBRIUM OF FORCES

- **A body** is in equilibrium if there is **no resultant force** and **no resultant torque**
- **The principle of moment:** for a body in **equilibrium**, the **sum of moments** at any point is **zero**

C. DENSITY AND PRESSURE

- **Density** is **mass per unit volume**

$$\rho = \frac{m}{V}$$

- **Pressure** is **force per unit area**

$$P = \frac{F}{A}$$

- For an object in a liquid, **the weight of the liquid above it** exerts a pressure on the object, known as the **hydrostatic pressure**
- It can be derived using the definitions above, resulting in:

$$P = \rho g h$$

Where:
P = Hydrostatic pressure (Pascal)
ρ = Density of the liquid (kg/m^3)
g = gravitational field strength (m/s^2)
h = depth (m)

- A **difference in hydrostatic pressure** can cause an object to **experience a force** known as **upthrust**

$$F = \rho g V$$

Where:
F = Upthrust (N)
ρ = Density of the liquid (kg/m^3)
g = gravitational field strength (m/s^2)
V = Volume of the submerged object/liquid displaced (m^3)

D. EXERCISE

1. **[9702_S21_qp_11_014]**
Water of depth 9.0 cm is covered by oil of depth 5.0 cm in a measuring cylinder. The density of the water is $1000\ kgm^{-3}$ and the density of the oil is $800\ kgm^{-3}$. What is the total pressure exerted on the base of the measuring cylinder due to the oil and water?

Solution
There are **two kinds of liquid** creating pressure on to the cylinder, so we need to calculate and **add them together**

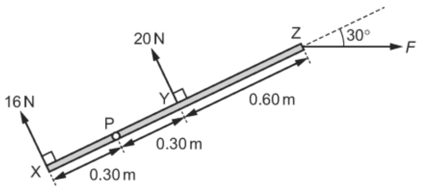
By using pressure formula with SI units,

$$P_w = \rho_w g h_w$$
$$P_w = 1000 \times 10 \times 9 \times 10^{-2}$$
$$P_w = 900$$

$$P_o = \rho_o g h_o$$
$$P_o = 800 \times 10 \times 5 \times 10^{-2}$$
$$P_o = 400$$

The total pressure on the cylinder is
900 + 400 = **$1300\ Pa$**

2. **[9702_S21_qp_11_013]**
A uniform rigid bar XZ with negligible mass is 1.20 m long. The bar is pivoted at point P. Three coplanar forces act on the bar as shown. Forces of 16 N and 20 N act perpendicularly to the bar at points X and Y respectively. Force F acts at point Z at an angle of 30° to the axis of the bar. The distances along the bar of the pivot and of the forces are shown



The bar experiences a resultant moment about P of 6.0 Nm in a clockwise direction

What is the magnitude of F?

Solution
First, calculate the known torques at **point X and Y to the pivot**

$$\tau_x = F_x d_x$$
$$\tau_x = 16 \times 0.3$$
$$\tau_x = 4.8\ clockwise$$

$$\tau_y = F_y d_y$$
$$\tau_y = 20 \times 0.3$$
$$\tau_y = 6\ anti-clockwise$$

The resultant of these **two torques** and the **torque from F** must **equal to 6 Nm** by assuming that the clockwise is positive

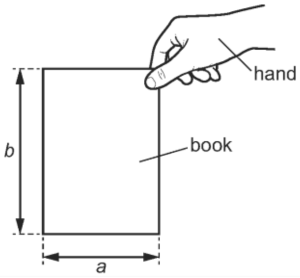
$$\tau = \tau_x + \tau_y + \tau_F$$
$$6 = 4.8 - 6 + \tau_F$$
$$\tau_F = 7.2$$

project the perpendicular to bar F

$$F \sin 30° \times d = 7.2$$
$$F \sin 30° \times 0.9 = 7.2$$
$$0.45\,F = 7.2$$
$$F = 16$$

The force on F is **16 N**

3. **[9702_S21_qp_13_014]**
A book of weight W has rectangular shape and is of uniform thickness. The book is held in a vertical plane so that the longer sides of the book are vertical, as shown



Which expression gives the approximate torque exerted by the hand on the book?

Solution
A torque will be generated from the **weight in the center** of the book, with the **perpendicular distance to the pivot** (hand) of 0.5a

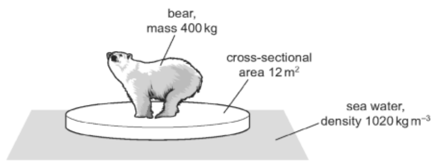
The magnitude is

$$\tau = F \times d$$
$$\tau = W \times \frac{1}{2} a$$
$$\tau = \frac{Wa}{2}$$

Because the **pivot is on the top right** of the book, it will **rotate anti-clockwise**.
That means the **hand will give the opposite direction which is clockwise**

4. **[9702_W20_qp_12_015]**

A cylindrical block of ice of cross-sectional area $12 m^2$ is floating, partially submerged, in the sea. The density of the sea water is $1020\ kgm^{-3}$. A polar bear of mass 400kg steps onto the block of ice



The block of ice sinks a vertical distance d.
What is the value of d?

Solution

The block of ice **is floating, so it is in equilibrium**. That means the force of the bear's **weight equals the upthrust**

$$W = F_u$$
$$mg = \rho V_{water}\ g$$
$$400 = 1020 \times 12 \times d$$
$$d = 0.032$$

The ice sinks at a distance of **3.3 cm**

5. **[9702_S20_qp_12_015]**

In a high-wire circus act, a man of mass 85kg is standing at rest at the midpoint of the wire.

the wire on either side of the man is at an angle of 20° to the horizontal.
What is the tension T in the wire?

Solution

The man is **at rest** so **the weight** of the man and the **vertical projection of the two tension** of the rope must be equal.

$$W = 2T \sin 20°$$
$$85 \times 10 = T \times 0.684$$
$$T = 1242.61$$

So the tension of the rope is **1.2 kN**