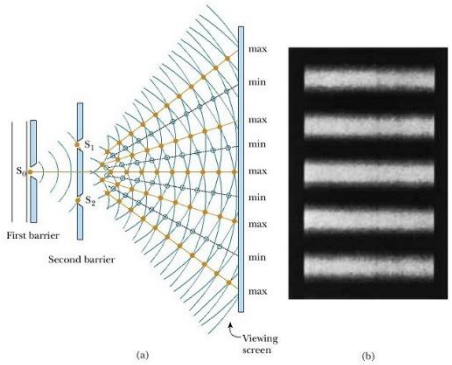
Superposition

A. Interference of waves

- **Interference** is where **two or more waves meet or overlap** to form a resultant wave.
- **The principle of superposition** dictates that the resultant displacement of the resultant wave from the interference is **equal to the sum of the displacements of the individual waves**
- **Constructive interference** happens when both waves arrive at a point **in phase.** If the original waves have equal amplitude, the resultant wave will have **twice the amplitude** of the original waves.
- **Destructive Interference** happens when both waves arrive at a point with **a phase difference of 180° or π radians**. If the original waves have equal amplitude, **the resultant wave will have zero amplitude**
- **An interference pattern** can be observed if the sources of interference are **coherent** (they maintain **a constant phase difference**) and they have **the same frequency**
- **Interference** can be observed in **Young's double-slit experiment**:



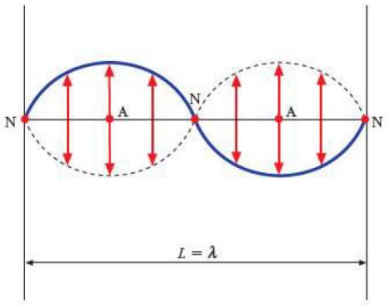
A bright fringe occurs every **nλ**
A dark fringe occurs every **(n+1/2)λ**
The relation between λ and fringe width is:

$$\lambda = \frac{ax}{D}$$

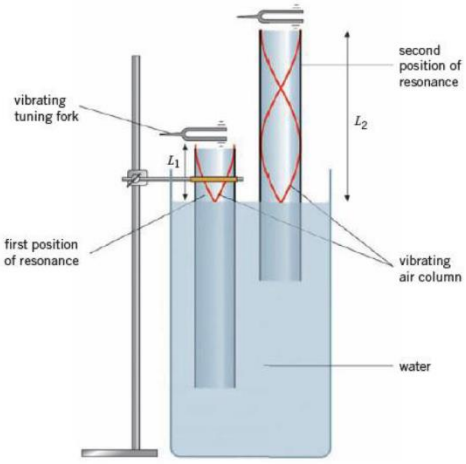
Where:
λ = Wavelength (m)
a = separation between slits (m)
x = fringe separation (m)
D = distance from 2nd slit to screen (m)

B. Stationary waves

- **A stationary wave** is formed from the interference of two waves with equal frequency, amplitude, and speed but **going in opposite directions.**
- A stationary wave **does not transfer energy**
- **A node** is a point in the stationary wave **which has zero amplitude**
- **An antinode** is a point in the stationary wave **which has the maximum amplitude**
- **Nodes and antinodes do not move** along the string
- Example of a stationary wave is a vibrating string that is attached to a wall or weight:

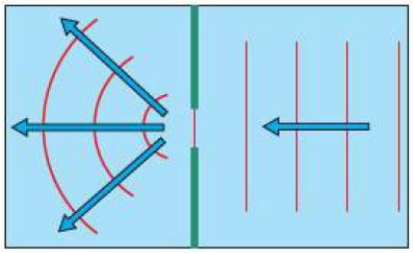


- Another example is when a tuning fork is held above an air column. At certain points, **the tube will vibrate with a loud sound** (resonance), this is the point **where a stationary wave is formed**.
- Node always forms at the **surface of the water**

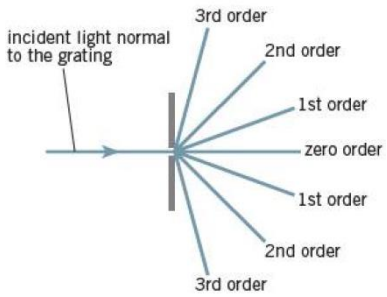


C. Diffraction and Diffraction grating

- **Diffraction** is the **spreading out of waves** when it passes **through a narrow gap**
- It is most noticeable **when the width of the gap is similar to the wavelength**



- Diffraction can be observed using a **diffraction grating**, which is a plate that contains a very large number of parallel, identical, very-closely-spaced slits.



- For a wave with **wavelength λ** passing through a diffraction grating with **slit separation d**, the following formula apply:

$$d \sin \theta = n\lambda$$

Where:
λ = Wavelength (m)
d = distance between slits(m)
θ = angle from horizontal
n = maxima order

D. EXERCISE

1) **[9702_s17_qp_11_028]**

A parallel beam of light of wavelength 600nm is incident normally on a diffraction grating. The grating has 300 lines per milimetre.

What is the total number of intensity maxima from the grating?

Answer:
First search for maxima order using the following formula : $d \sin\theta = n\lambda \; and \; d = \frac{1}{N}$
Remember to convert them to SI

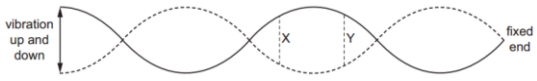
$$d \sin\theta = n\lambda$$
$$\frac{1}{\frac{300 lines}{mm}} \times \sin 90° = n \times 600 \times 10^{-9}$$
$$\frac{1 \times 10^{-2} \times 10^{-3}}{3} \times 1 = 6 \times 10^{-7} \times n$$
$$\frac{100}{18} = n$$
$$n = 5{,}56$$

Round the amount down to 5, because we dont count the sixth order unless it fully reveals itself

Remember that there is the **zeroth order in the middle, then 5 orders upwards and 5 orders downwards.**
So there is 11 total intensity maxima

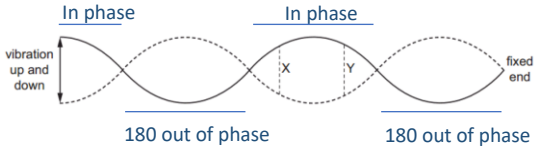
2) **[9702_w16_qp_11_028]**

The diagram shows a long rope fized at one end. The other end is moved up and down, setting up a stationary wave.



What is the phase difference between the oscillations at X and Y?

Answer:
All of the particles between two adjacent nodes **are in phase with eachother, but 180 degrees out of phase to other particles** between the next set vof nodes before or after them.

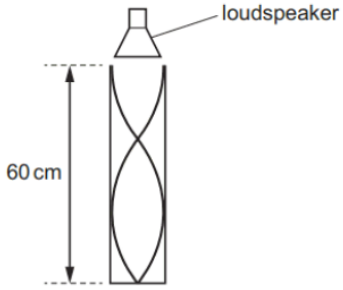


Because at the next instance, the ones that are **in phase would all oscillate downwards** while the **180 out of phase ones would all oscillate upwards.**

So there is 0 degrees phase difference

3. **[9702_w18_qp_12_024]**

The sound from a loudspeaker placed above a tube causes resonance of the air in the tube. A stationary wave is formed with two nodes and two antinodes as shown.



The speed of sound in the air is 340 m/s.
What is the frequency of the sound?

Answer:
The wave inside is $\frac{3}{4}\lambda$ long, so the following applies:

$$\frac{3}{4}\lambda = 0.6m$$
$$\lambda = 0.8m$$

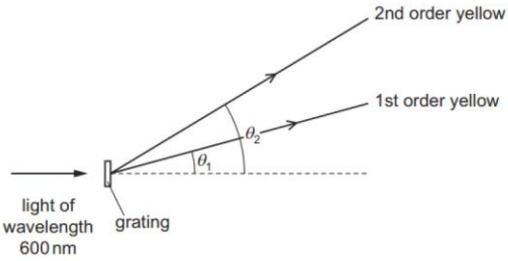
Then find the frequency by the following formula:

$$v = \lambda f$$
$$340 = 0.8 \times f$$
$$f = 425$$

So the frequency is 425 Hz

4. **[9702_w14_qp_11_027]**

A diffraction grating experiment is set up using yellow light of wavelength 600nm. The grating has a slit seperation of $2.00\mu m$.



What is the angular seperation $(\theta_2 - \theta_1)$ between the first and the second order maxima of the yellow light?

Answer:
Search for both angles using the following formula:

Remember to convert them to SI units!

$$d \sin \theta = n\lambda$$
$$2 \times 10^{-6} \sin \theta = 1 \times 600 \times 10^{-9}$$
$$\sin \theta = 0.3$$
$$\theta_1 = 17.457$$

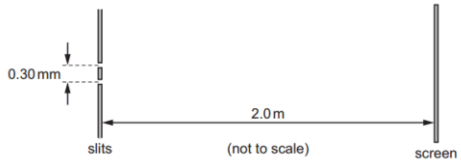
$$d \sin \theta = n\lambda$$
$$2 \times 10^{-6} \sin \theta = 2 \times 600 \times 10^{-9}$$
$$\sin \theta = 0.6$$
$$\theta_2 = 36.869$$

$$\theta_2 - \theta_1 = 36.869 - 17.457$$
$$\theta_2 - \theta_1 = 19.412°$$

So the angular seperation is 19.4 degrees

5. **[9702_w16_qp_12_028]**

Monochromatic light of wavelength 450nm passes through two parallel slits 0.30mm apart. Bright fringes are observed on a screen 2.0m away.



How far apart are the bright fringes on the screen?

Answer:
Use the following formula and convert the data to SI units.

$$\frac{\lambda}{a} = \frac{x}{D}$$
$$\frac{450 \times 10^{-9}}{0.3 \times 10^{-3}} = \frac{x}{2}$$
$$x = 3 \times 10^{-3}$$

So the distance between the bright fringes is 3 mm