Discrete Random Variables

A. DEFINITION

- A *discrete random variable* is one which may take on only a countable number of distinct values such as 0,1,2,3,4 and so on. Discrete random variables are usually (but not necessarily) counts. If a random variable can take only a finite number of distinct values, then it must be discrete
- Examples of discrete random variables include the number of children in a family, the Friday night attendance at a cinema, the number of patients in a doctor's surgery, the number of defective light bulbs in a box of ten

B. NOTATION AND CONDITIONS FOR A DISCRETE RANDOM VARIABLE

- A discrete random variable is usually denoted by an uppercase letter, such as $X, Y,$ or Z
- The particular value that the variable takes are denoted by lower case letters, such as r. Sometimes these are given suffixes $r_1, r_2, r_3, \dots$
- P $(X = r_1)$ is the probability that the discrete random variable X takes the particular value r_1. The expression P $(X = r)$ is used to express a more general idea
- For a discrete random variable, X, which can take only the values $r_1, r_2, \dots, r_n$ with probabilities $p_1, p_2, \dots, p_n$

$$p_1 + p_2 + \cdots + p_n$$

$$\sum_{i=1}^{n} p_i = \sum_{i=1}^{n} \mathrm{P}\,(X = r_1) = p_r = 1$$

C. PROBABILITY DISTRIBUTION TABLES

- Expectation

$$E(X) = \mu = \sum x_i\, p_i$$

- Expectation squared

$$E(X^2) = \sum (x_i)^2\, p_i$$

- Variance

$$\sigma^2 = \sum (x_i - \mu)^2\, p_i = \sum x_i^2 p_i - \mu^2$$

D. EXERCISE

A factory makes a large number of ropes with lengths either 3m or 5m. There are four times as many ropes of length 3m as there are ropes of length 5m. One rope is chosen at random. Find the expectation and variance of its length!

Solution:

$$P(3m\ Rope) = \frac{4}{5}$$

$$P(5m\ Rope) = \frac{1}{5}$$

Expectation / mean

$$E(X) = \sum x_i\, p_i = \left(3 \times \frac{4}{5}\right) + \left(5 \times \frac{1}{5}\right) = 3.4$$

Expectation squared

$$E(X^2) = \sum (x_i)^2\, p_i$$

$$\left(3^2 \times \frac{4}{5}\right) + \left(5^2 \times \frac{1}{5}\right) = 12.2$$

Variance

$$\sigma^2 = \sum x_i^2 p_i - \mu^2 = 12.2 - (3.4)^2 = 0.64$$