Momentum

- Defined as the product of its mass and velocity

$$\vec{p} = m\vec{v}$$

- Unit: Newton second (Ns)

A. CONSERVATION OF LINEAR MOMENTUM

- From Newton's 1st law, momentum of an object is constant, provided the external resultant force is zero
- From Newton's 2nd law, $F_{net} = \frac{\Delta p}{\Delta t} = \frac{\Delta(mv)}{\Delta t}$. Resultant force is the rate of change of momentum.
- From Newton's 3rd law $F_{action} = -F_{reaction}$. Considering two isolated objects exerting force on each other.

$$F_{2on1} = -F_{1on2}$$

$$\frac{\Delta p_1}{\Delta t} = -\frac{\Delta p_2}{\Delta t}$$

$$\text{since}\Delta t \text{ is equal for both particles, } \Delta p_1 = -\Delta p_2$$

$$m_1v_1 - m_1u_1 = -(m_2v_2 - m_2u_2)$$

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

- The total momentum of the 2 bodies finally is the same as the total momentum of the 2 bodies initially. Hence, the momentum of the system is conserved given that there is no external **net** force.
- **What if there are external forces?** If external forces are considered, as long as they add up to zero (zero net force), the momentum of the 2 body-system will still be conserved.

B. IMPULSE

- Considering a constant force F acts on an object for time Δt, then impulse of the force is $F\Delta t$

Since $F\Delta t = \Delta p$ (Newton's 2nd law),

Impulse is equal to change in momentum.

- Unit: Newton second (Ns) or $kg\ m\ s^{-1}$

C. ELASTIC COLLISIONS

Linear Momentum and it's Conservation

- The principle of conservation of momentum states that **momentum is always conserved** in any interaction where **no external forces act.**
- The kinetic energy before collision is **equal** to the kinetic energy after collision.

$$m_av_a + m_bv_b = m_av_a' + m_bv_b'$$

$$Total\ KE\ before = Total\ KE\ after$$

$$\frac{1}{2}m_av_a^2 + \frac{1}{2}m_bv_b^2 = \frac{1}{2}m_av_a'^2 + \frac{1}{2}m_bv_b'^2$$

Object A collides with an equal mass target B which is at rest:

- The impacting object comes to a dead stop, the target gains the exact same speed as the impacting object.

$$v_a' = 0, \qquad v_b' = v_a$$

Object A collides with an equal mass object B. Objects have equal but oppositely directed velocity:

- The two objects bounce off each other, exchanging velocity. Interestingly, this result also holds for two objects colliding with equal but opposite momentum: the objects will swap momentum. This is a very useful result which allows us to simplify otherwise complex elastic collision problems.

$$v_a' = v_b, \qquad v_b' = v_a$$

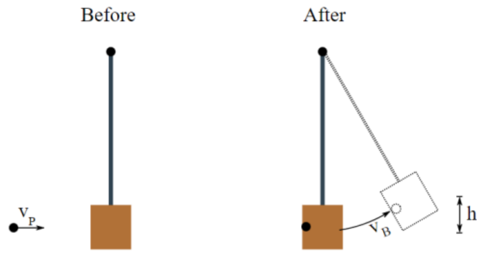
A heavy object collides with a much lighter target which is at rest.

- The final velocity of the heavy object tends to its initial velocity. This is fairly intuitive; the light object has little effect on the heavy one.

A light object collides with a much heavier target which is at rest.

- The light object bounces off the target, maintaining the same speed but with opposite direction. The heavy target remains at rest.

D. INELASTIC COLLISIONS



Here we use the subscript B for the block, P for the projectile. v_B is the velocity of the block after the impact.

$$v_P = \frac{m_P + m_B}{m_P}\sqrt{2gh}$$

C. COLLISIONS IN TWO DIMENSIONS

- If two objects make a glancing collision, they will move off in two dimensions after the collision (e.g. a glancing collision between two billiard balls)
- For a collision where objects will be moving in two dimensions, **the momentum will be conserved in each direction independently** as long there is no external impulse in that direction

The total momentum in x direction will be the same before and after collision.

$$\Sigma p_{xi} = \Sigma p_{xf}$$

The total momentum in y direction will be the same before and after collision.

$$\Sigma p_{yi} = \Sigma p_{yf}$$

- How to solve two dimensional collision problems:
  - Identify all the bodies in the system
  - Write down all the values you know and what you need to find out
  - Identify all the forces acting on each of the bodies. Remember that conservation of momentum only applies if there is no external impulse. However, conservation of momentum can be applied separately to horizontal and vertical components.
  - Write down the equations which equate momentum before and after collision, and solve the equations to determine the expression you need to solve.
  - Substitute in the numbers you know to find the final value.

E. EXERCISE

8 A golf ball of mass m is dropped onto a hard surface from a height h_1 and rebounds to a height h_2.

The momentum of the golf ball just as it reaches the surface is different from its momentum just as it leaves the surface.

What is the total change in the momentum of the golf ball between these two instants? (Ignore air resistance.)

A $m\sqrt{2gh_1} - m\sqrt{2gh_2}$

B $m\sqrt{2gh_1} + m\sqrt{2gh_2}$

C $m\sqrt{2g(h_1 - h_2)}$

D $m\sqrt{2g(h_1 + h_2)}$

Solution:

$$\Delta p = mv - mu$$

Considering any upward movement as positive.

By energy conservation before the bounce, and after the bounce.

$\frac{1}{2}mu^2 = mgh_1$. Hence, $u = \sqrt{2gh_1}$

$\frac{1}{2}mv^2 = mgh_2$. Hence, $v = \sqrt{2gh_2}$

$m\sqrt{2gh_2} - (-\sqrt{2gh_1}) = m\sqrt{2gh_2} + m\sqrt{2gh_1}$

The minus sign accounted for the downward movements of the golf ball when falling.

Answer: B

7 The mass of a rocket-propelled truck is approximately equal to the mass of the fuel in its tank. The fuel is ignited and the truck is propelled along horizontal tracks by a constant force. The effect of air resistance is negligible.

During a test run the fuel is consumed at a constant rate.

Which statement describes the acceleration of the truck during the test run?

A The acceleration of the truck decreases as the fuel is consumed.

B The acceleration of the truck increases as the fuel is consumed.

C The acceleration of the truck remains constant.

D The acceleration of the truck is zero and the truck moves at a constant velocity.

Solution:

Although the rockets propelled with a constant force, the mass decreases as the rocket moves. Hence, the acceleration is increasing instead of remaining constant.

Another approach is by newton's 2nd law. But in this case, the mass is not constant.

$$F = \frac{dp}{dt} = \frac{d}{dt}(mv)$$

By product rule,

$$F = \frac{dm}{dt}v + m\frac{dv}{dt}$$

$$F = \frac{dm}{dt}v + ma$$

We already know that $\frac{dm}{dt}$ is decreasing and v must be positive.Then, to make F constant, $\frac{dv}{dt}$ must be increasing.

Answer: B