Dynamics

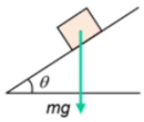
A. NEWTON'S LAW OF MOTION

- **First law**: Every body continues in its state of rest or of uniform motion in a straight line until it is caused to change by a net force or torque.
    - If we remove all forces acting on a body, it will stay at rest if it were originally at rest and it will move with constant speed in a straight line if that was what it was doing before.
    - Also called the law of inertia, that is a tendency to resist change. Inertia is dependent on mass. Hence, a more massive object is more likely to stay at rest. Similarly, it has a greater tendency to maintain its velocity, or, more force is required to change its velocity (both magnitude and direction)
- **Second law**: The rate of change of momentum of a body is proportional to the resultant force and occurs in the direction of force.
    - Change of momentum(vector) must be in the same direction as the net force(vector).

$$\vec{F}_{Net} = \frac{d\vec{p}}{dt} \qquad \vec{F}_{Net} = m\frac{d\vec{v}}{dt} = m\vec{a}$$

$$Avg\ net\ force, \vec{F}_{Net} = \frac{\Delta\vec{p}}{\Delta t}$$

- Consider a box of mass m sliding down a rough surface that is inclined by the angle of θ and coefficient of friction μ



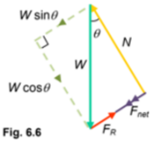


Fig. 6.6

Since there is a frictional force, Normal force is worth to consider as shown on the right fig.

$$F_{Net} = ma$$

$$\frac{W sin\theta - \mu W cos\theta}{m} = a$$

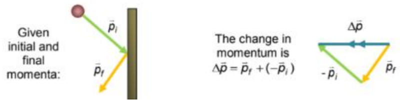
$$g sin\theta - \mu g cos\theta = a$$

- **Third law**: If a body A exerts a force on a body B, then body B exerts an equal but opposite force on body A, forming an action-reaction pair
    - If the earth pulls on a person with a gravitational force, the person must be pulling on Earth with the same type of force.
    - Important notes for action-reaction pair of forces:
        a) they have equal magnitude
        b) they have opposite directions
        c) they act on different bodies
        d) they are of the same type

B. MOMENTUM

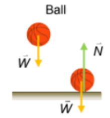
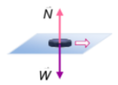
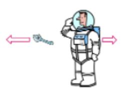
Defined as the product of its mass and velocity; $\vec{p} = m\vec{v}$

- Consider a billiard ball collides with side of the pool, the change in momentum



The $\Delta\vec{p}$ is the direction of the net force acting on the ball.

- Conservation of momentum: when bodies in a system interact, total momentum remains constant provided no external force acts on the system.
- From $\vec{F}_{Net} = \frac{d\vec{p}}{dt}$, it follows that the change in momentum is zero.
- Equation: $m_a u_a + m_b u_b = m_a v_a + m_b v_b$
- Applications:

Situation	System	System's momentum
A ball bouncing on the floor.	Ball 	Ball's **momentum is not constant** because there is mostly **a net force acting on it**. Net force is weight when ball is in the air. Net force is the resultant of the normal contact force and weight when on floor.
An ice hockey puck sliding horizontally across the ice rink at constant velocity.	Puck 	Assume no friction. **No net force** both vertically and horizontally so the puck moves with **constant momentum**.
An astronaut throws a spanner.	Astronaut and spanner 	Assume astronaut-spanner system **does not experience a net force** such as gravitational pull from a nearby planet. Though the astronaut and spanner individually undergoes a change in momentum during the throw, the **total momentum of the system is constant at all times**.

C. WEIGHT AND MASS

Mass (Kg)	**Weight** (Newtons)
Property of a body which resists change in motion	Property of a body which resists change in motion; W=mg
Scalar quantity	Vector quantity
Constant throughout the universe	Not constant, depending on gravitational acceleration

- Consider a person standing on a bathroom scale inside a moving elevator as shown below.
    - The elevator is moving upwards and slowing down with a deceleration a. Scale will measures the normal force which is equal but opposite to N
      Hence, W: weight in normal condition and N: weight inside the accelerated elevator.
    - In this case, the N must be smaller than W otherwise the person cannot be accelerated downwards.
    - $F_{Net} = ma$
      $mg - N = ma$
      $N = m(g - a)$
      $N < W$, the person "loss" weight

❖ Similarly, if it is accelerated upwards, the person will gain weight

❖ From the equation, we could also know that if the elevator's acceleration is g (which will happen if the cable is cut down), the scale will reads zero and the person is considered to be weightless!

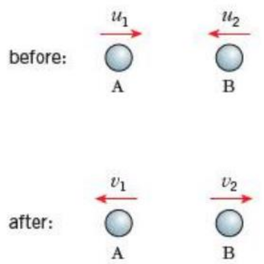
D. ELASTIC COLLISIONS

Linear Momentum and it's Conservation

- The principle of conservation of momentum states that **momentum is always conserved** in any interaction where **no external forces act.**
- The kinetic energy before collision is **equal** to the kinetic energy after collision.

$$Total\ KE\ before = Total\ KE\ after$$

$$\frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{1}{2}m_{1'} v_{1'}^2 + \frac{1}{2}m_{2'} v_{2'}^2$$



- When the velocity directions are as defined in the picture, application of the two conservation conditions shows that the relative speed of approach $(u_1 + u_2)$is equal to the relative speed of separation $(v_1 + v_2)$

$$u_1 + u_2 = v_1 + v_2$$

This formula only applies only for a perfectly elastic collision.

E. INELASTIC COLLISIONS

- Where only momentum is conserved, while some of the kinetic energy is converted into other form (heat, sound, etc) and may be larger or smaller after a collision.
- Although kinetic energy may or may not conserved in a collision, **momentum is always conserved, and so is total energy.**
- **Perfectly inelastic collision:** only momentum is conserved, and the particles stick together after collision

C. COLLISIONS IN TWO DIMENSIONS

- If two objects make a glancing collision, they will move off in two dimensions after the collision (e.g. a glancing collision between two billiard balls)
- For a collision where objects will be moving in two dimensions, **the momentum will be conserved in each direction independently** as long there is no external impulse in that direction

- The total momentum in x direction will be the same before and after collision.

$$\Sigma p_{xi} = \Sigma p_{xf}$$

- The total momentum in y direction will be the same before and after collision.

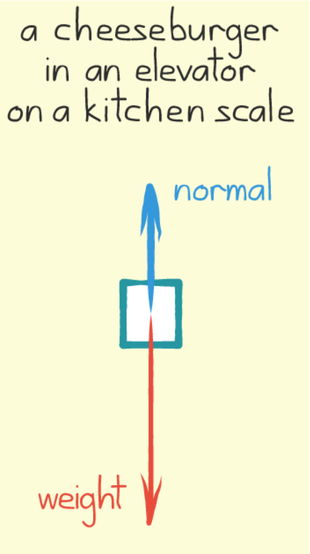
$$\Sigma p_{yi} = \Sigma p_{yf}$$

- How to solve two dimensional collision problems:
  - Identify all the bodies in the system
  - Write down all the values you know and what you need to find out
  - Identify all the forces acting on each of the bodies. Remember that conservation of momentum only applies if there is no external impulse. However, conservation of momentum can be applied separately to horizontal and vertical components.
  - Write down the equations which equate momentum before and after collision, and solve the equations to determine the expression you need to solve.
  - Substitute in the numbers you know to find the final value.

D. EXERCISE

- A person stands in an elevator weighing a cheeseburger with a kitchen scale. The mass of the cheeseburger is 0.150 kg. the scale reads 1.14 N
  (i). determine the magnitude and direction of the net force on the cheeseburger.
  (ii) determine the magnitude and direction of the elevator acceleration.

solutions



(i). find the weight first

W=mg

W=(0.150 kg)(9.8 m/s^2)

W=1.47 N

The net force is downward, so

$\Sigma F = N - W$

$\Sigma F = 1.14N - 1.47N$

$\Sigma F = -0.33N\ down$

(ii) use the Newton's second law of motion to determine the acceleration.

$a = \frac{\Sigma F}{m}$

$a = \frac{-0.33\text{N}}{0.150\ kg}$

$a = -2.2\frac{m}{s^2}\ down$