Kinematics

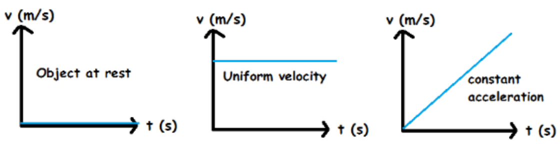
A. MOTION

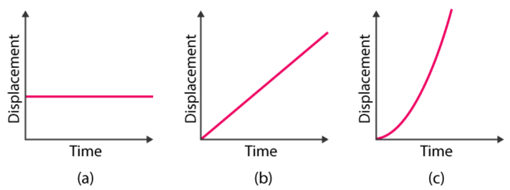
Quantity	Definition	Symbol/Type
Distance	The length of the actual path travelled by an object.	$s/scalar$
Displacement	Straight line distance in a specific direction.	$\bar{s}/vector$
Speed	Rate of change of distance.	$v/scalar$
Velocity	Rate of change of displacement.	$\bar{v}/vector$
Acceleration	Rate of change of velocity.	$\bar{a}/vector$

- A scalar is a quantity which only has a magnitude but no direction
- A vector is a quantity which has a magnitude and a direction.

Some WRONG definitions:

- Speed is the rate of change of distance per unit time - wrong because rate of change already implies per unit time.
- Velocity is the rate of change of displacement per unit time - wrong because rate of change already implies per unit time.





Note:

- Speed and velocity are the slope of the distance time graph and displacement time graph respectively.
- Area under the velocity time graph is the displacement of the object.

B. EQUATIONS OF MOTION

- This equation will be valid if the acceleration of an object is **constant/uniform.**

$$v^2 = u^2 + 2as$$

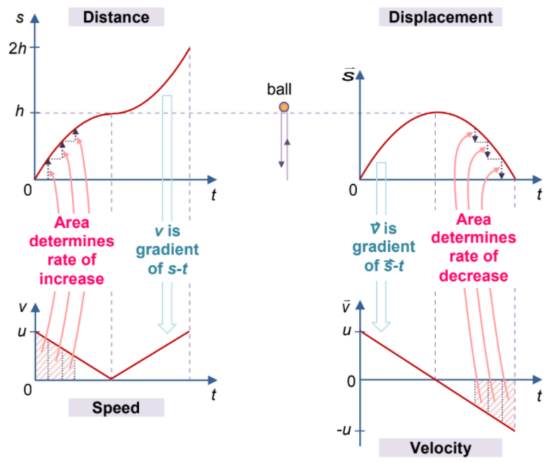
$$v = u + at$$

$$s = u^2 + \frac{1}{2}at^2$$

$$s = \frac{(u+v)}{2}t$$

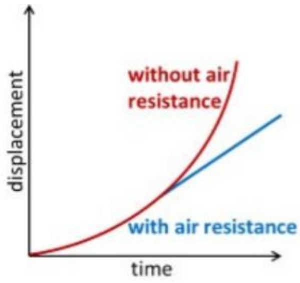
u= initial velocity
a= acceleration
v= final velocity
s= displacement

- For a projected ball's journey up and down, the first two plots below show the difference between distance-time and displacement-time graphs.
- The bottom two plots show the difference between the speed-time and velocity- time graphs.



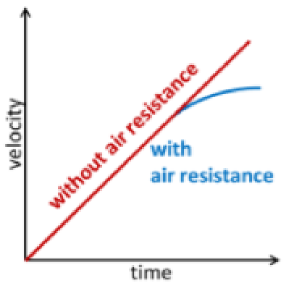
C. MOTION OF BODIES FREE FALLING

Displacement



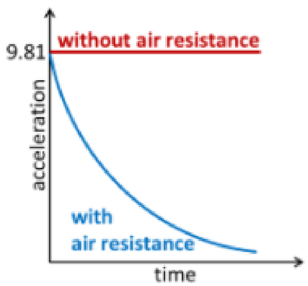
- **Red line:** Continues to curve as it accelerate
- **Blue line:** Graph levels off as it reaches terminal velocity

Velocity



- **Red line:** Continues to accelerate constantly
- **Blue line:** Graph curves as it decelerates and levels off to terminal velocity

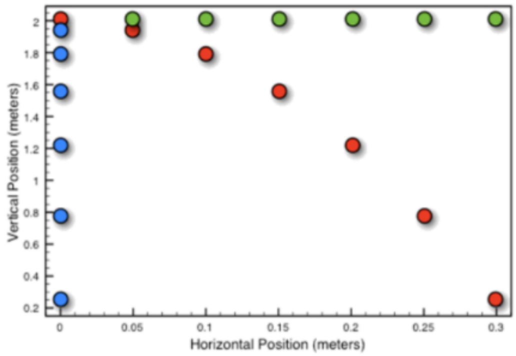
Acceleration



- **Red line:** Straight line
- **Blue line:** Graph curves down to zero because resultant force equals to zero

D. PROJECTILE MOTION

- **Projectile motion:** is the motion of an object thrown or projected into the air.
- **Projectile:** the object
- **Trajectory:** its path



- **Horizontal motion:** Constant velocity (speed at which projectile is thrown)
- **Vertical motion:** Constat acceleration (cause by weight of object, constant free fall acceleration)
- **Curved path** – parabolic ($y \propto x^2$)

For finding different parameters related to projectile motion, we can make use of differential **equations of motions**

Total Time of Flight (t)

Resultant displacement (s) = 0 in vertical direction

$$t = \frac{2u\,sin\theta}{g}$$

Horizontal Range (R)

Horizontal range (OA) = horizontal component of velocity (ux) x total flight time (t)

$$R = \frac{u^2 sin2\theta}{g}$$

Maximum Height (H_{max})

It is the highest point of the trajectory

$$H_{max} = \frac{u^2 \sin^2\theta}{2g}$$

The Equation of Trajectory

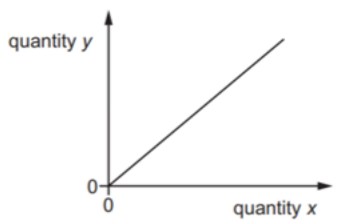
$$Equation\ of\ Trajectory = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

Component of Velocity

	Horizontal	Vertical
Without air resistance	Constant	Increases at a constant rate
With air resistance	Decreases to zero	Increases to a constant value

E. EXERCISE

The graph shows the variation of a quantity y with a quantity x for a body that is failing in air at constant (terminal) velocity in a uniform gravitational field.



Which quantities could x and y represent?

	x	Y
A	Air resistance	Acceleration
B	Loss of height	Gain in kinetic energy
C	Loss of potential energy	Work done against air resistance
D	Time	Velocity

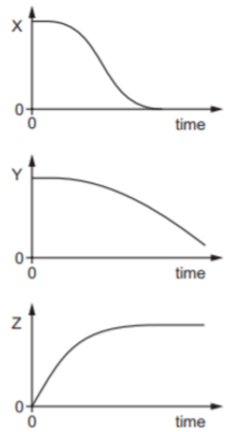
Solution:

- **A is incorrect.** The downward force for of falling object is gravity and the upward force is air resistance. Hence if the air resistance is increasing, the acceleration will therefore decrease.
- **B is incorrect.** It's clearly shown with Y. It is because, as terminal velocity is reached by an object, the acceleration will be equal to zero, hence no change in velocity, hence the kinetic energy will be constant.
- **D option is incorrect** because as mentioned earlier, the velocity of an object will be constant as it reaches the terminal velocity.
- **C is correct** since there is no change in KE, hence the loss in PE when the ball is falling will be equal to the work done against the external force (in this case air resistance). Note that if the object has not reached the terminal velocity yet, this equation can be implies: $Loss\ is\ PE = W_{external} + gain\ in\ KE$

Answer: C

An object is dropped at time t= 0 from a high building. Air resistance is significant. Three graph are plotted against time.

- The height of the object above the ground.
- The speed of the object
- The magnitude of the resultant force on the object



What are the quantities X, Y and Z?

	Height of the object above the ground	Speed of the object	Magnitude of the resultant force on the object
A	X	Y	Z
B	X	Z	Y
C	Y	Z	X
D	Z	Y	X

Solution:

- Since the air resistance is significant, hence the object will reach the terminal velocity. First, you can notice that the Z-time graph is a very common graph when describing an object in a terminal velocity, and it is a good start in answering the question.
- The Z-time graph is the velocity-time graph. The object will have a constant acceleration g at the beginning of the motion since it undergoes a free fall. As the air resistance takes place, the velocity will decrease before eventually remaining constant when the air resistance is equal to g. In this case there will be no significant different between velocity and speed.
- The X-time graph is the resultant force of the object. At the terminal velocity, the air resistance is equal to gravity (g), hence resultant force is zero. And the only graph which touches the $x\ axis$ is the X-time graph.
- The Y-time graph is the height of the object. This reason is, at the beginning of the motion or at the free fall, the velocity of the ball will increase with constant acceleration, hence the height will decline with an increasing gradient. At the terminal velocity, acceleration is zero and the velocity is constant, hence the height will decline with a constant gradient (which is shown at the very end of the graph).

Answer: C