Oscillation

A. DESCRIPTION

- **Oscillations:** A repetitive or cyclical variation of a quantity while vibration is subset of oscillation involving mechanical systems.
- **Restoring Force:** a force that always try to bring an oscillating body back to its equilibrium position whenever it is displaced from that equilibrium position.
- **Amplitude (x_0):** The maximum displacement of the oscillating mass from the equilibrium position in either direction
- **Equilibrium Position:** the natural position of the oscillating mass where there is no net force acts on it

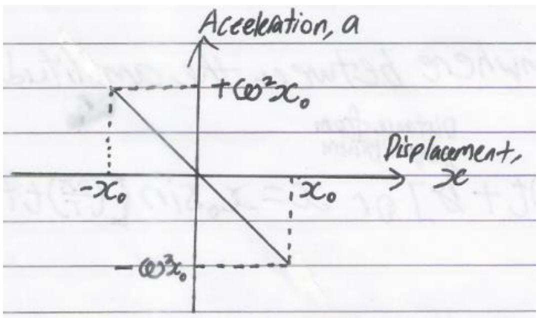
B. SIMPLE HARMONIC MOTION

- **SHM** is an oscillatory or periodic motion whereby the acceleration is proportional to the displacement from the equilibrium position and in opposite direction or always towards the equilibrium position.
- Mathematically the definition is

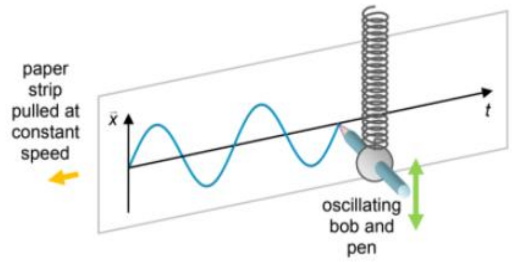
$$a = -\omega^2 x$$

Where ω is angular velocity/angular frequency and equal

$$\omega = \frac{2\pi}{T} = 2\pi f$$

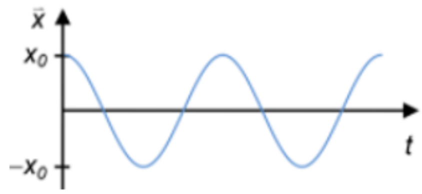


C. MOTIONS



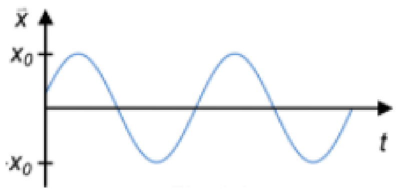
- if the pencil start plotting when the spring is at the equilibrium position,
  $t = 0, x = 0$

$$x = x_0 sin\omega t$$



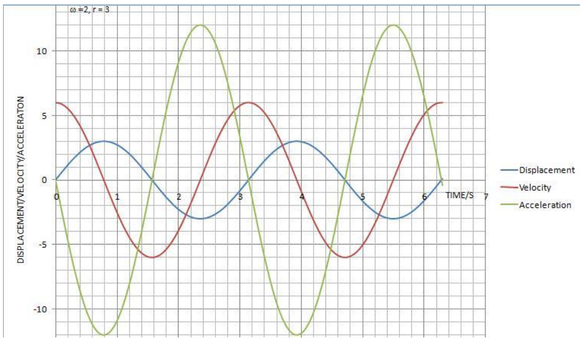
- if the pencil start potting when the spring is at the highest position, $t = 0, x = x_0$

$$x = x_0 cos\omega t$$



$x_0 = amplitude$

- Since $v = \frac{dx}{dt}$ and $a = \frac{d^2x}{dt}$,



$v = \frac{d}{dt}(x_0 sin\omega t)$	$v = \omega \cdot x_0 cos\omega t$
$a = \frac{d}{dt}(x_0 \omega cos\omega t)$	$a = -x_0\omega^2\ sin\omega t$

$v = \frac{d}{dt}(x_0 cos\omega t)$	$v = -\omega \cdot x_0 sin\omega t$
$a = \frac{d}{dt}(-x_0 \omega sin\omega t)$	$a = -x_0\omega^2\ cos\omega t$

Note: the graph illustrated the $x = x_0 sin\omega t$ equation and its derivations.

From the graph we can conclude that

- Maximum velocity occurred when the particle is on its equilibrium position and dropped to zero when the particle has its maximum displacement.
- Maximum acceleration occurred in opposite when the particle has its maximum displacement and zero at equilibrium

D. ENERGY IN SHM

In SHM, the maximum kinetic energy occurred when particle is at equilibrium position because the velocity is maximum.

$$E_{total} = PE_{max} = KE_{max}$$

$$KE_{max} = \frac{1}{2}m(v_{max})^2$$

- Since v_{max} is $x_0\omega$, hence

$$KE_{max} = \frac{1}{2}mx_0^2\omega^2$$

- We already know that

$\sin^2\theta + \cos^2\theta = 1$ and

$x = x_0 sin\omega t$ and

$v = \omega \cdot x_0 cos\omega t$

$sin\theta = sin\omega t = \frac{x}{x_o}$ and

$cos\theta = \cos\omega t = \frac{v}{\omega x_0}$

$$\frac{x^2}{x_0^2} + \frac{v^2}{(\omega x_0)^2} = 1$$

$$x^2\omega^2 + v^2 = \omega^2 x_0^2$$

$$v^2 = \omega^2(x_0^2 - x^2)$$

$$v = \pm\omega\sqrt{(x_0^2 - x^2)}$$

- Hence, at a given displacement,
  - **Kinetic Energy**

$$\frac{1}{2}m\omega^2(x_0^2 - x^2)$$

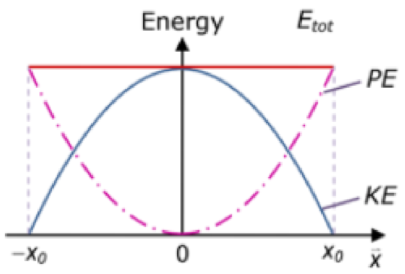
  - **Potential Energy**

$$E_{total} - KE$$

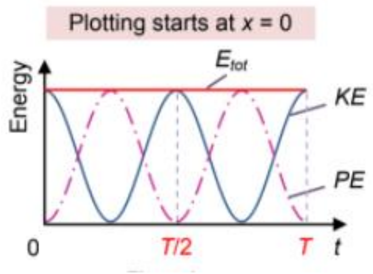
$$\frac{1}{2}mx_0^2\omega^2 - \frac{1}{2}m\omega^2(x_0^2 - x^2)$$

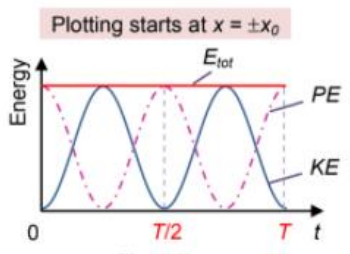
$$\frac{1}{2}\omega^2x^2$$

- **The E-x graph**



- **The E-t graph**



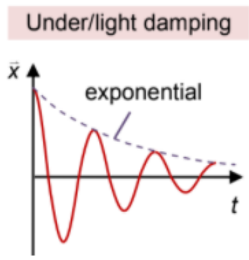


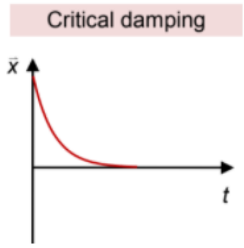
Hence, for a given time,

$v = \omega \cdot x_0 cos\omega t$
$$KE = \frac{1}{2} m\omega^2 x_0^2 \cos^2 \omega t$$ $$PE = \frac{1}{2} m x_0^2 \omega^2 - \frac{1}{2} m\omega^2 x_0^2 \cos^2 \omega t$$ $$= \frac{1}{2} m\omega^2 x_0^2 (\sin^2 \omega t)$$
$v = -\omega \cdot x_0 sin\omega t$
$$KE = \frac{1}{2} m\omega^2 x_0^2 \sin^2 \omega t$$ $$PE = \frac{1}{2} m x_0^2 \omega^2 - \frac{1}{2} m\omega^2 x_0^2 \sin^2 \omega t$$ $$= \frac{1}{2} m\omega^2 x_0^2 (\cos^2 wt)$$

E. DAMPING

Loss of energy and reduction in amplitude from an oscillating system caused by resistive force (force acting in opposite direction to the motion).





- **Light damping**
  - Oscillates about equilibrium with gradually decreasing in amplitude
  - The damping is less than critical damping.

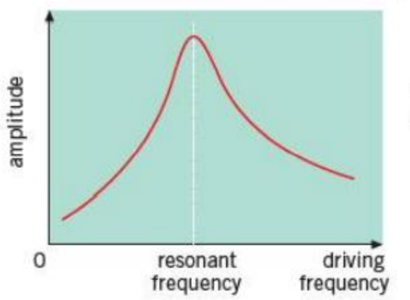
- **Critical damping**
  - Displacement reduced to zero in shortest possible time without oscillation.
  - Used in car suspension, to minimize oscillations yet not over damped till too stiff for comfort. Also used in machines, buildings, and bridges to remove vibrations.

- **Heavy damping**
  - Reducing the displacement to zero exponentially without oscillation but in a longer time than critical damping (very-very slow).
  - The damping is greater than critical damping.
  - Used in auto-closing door so that door closes gently.

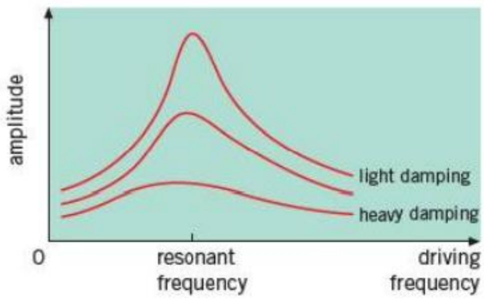
F. FORCED OSCILLATIONS AND RESONANCE

- When a vibrating body undergoes free (undamped) oscillations, it vibrates at its **natural frequency.**
- **Forced oscillations** refer to the application of a periodic driving force to force an oscillator to oscillate at the frequency of the driving force.
- **Resonance** occurs when the natural frequency of vibration of an object is equal to the driving frequency, giving a maximum amplitude of vibration.

- **Resonance curve**



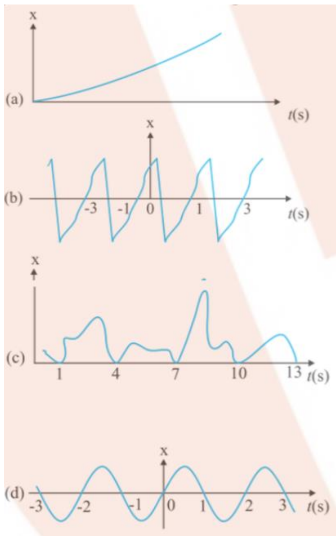
- **Effect of damping on the resonance curve**



- Effects of damping on frequency response of a system undergoing forced oscillations:
  - Decreases amplitude at all frequencies.
  - Slightly decreases resonant frequency.
  - Resonant peak becomes flatter.

G. EXERCISE

- Figure depicts four x-t plots for linear motion of a particle. Which of the plots represent periodic motion? What is the period of motion (in case of periodic motion)?



Solution:
(a) It is not a periodic motion. This represents a unidirectional, linear uniform motion. There is no repetition of motion in this case.
(b) In this case, the motion of the particle repeats itself after 2s . Hence, it is a periodic motion, having a period of 2s.
(c) It is not a periodic motion. This is because the particle repeats the motion in one position only. For a periodic motion, the entire motion of the particle must be repeated in equal intervals of time.
(d) In this case, the motion of the particle repeats itself after 2 s. Hence, it is a periodic motion, having a period of 2 s.

- An 8 kg body performs S.H.M. of amplitude 30 cm. The restoring force is 60N, when the displacement is 30cm. Find the Time period.

Solution:

m = 8 kg

A = 30 cm = 0.30 m

X = 30 cm = 0.30 m

f = 60 N

k = spring constant

since, f = kx

$$k = \frac{f}{x} = \frac{60}{0.30} = 200\ N/m$$

As, Angular velocity

$$w = \sqrt{\frac{k}{m}} = \sqrt{\frac{200}{8}} = 5\ s$$

Time period, $T = \frac{2\pi}{w} = \frac{2\times22}{7\times5} = 1.256\ s$

- A body performing simple harmonic motion has a displacement x given by the equation $x = 9sin45t$, where t is the time in seconds. What is the frequency of the oscillations?

Solutions:

Since $2\pi f = \omega$ and from the equation of displacement, we can get $\omega = 40$ then the frequency is

$$f = \frac{20}{\pi}$$

$$f = 6{,}366\ Hz$$