Probability

A. MEASURING PROBABILITY

Probability (a change) is a way of describing the likelihood of different possible *outcome* occuring as a result of some *experiment*.

It is important in probability to distinguish experiments from the outcomes which they may generate. Here are few examples.

Experiment	Possible Outcomes
Guessing athe answer a four-option multiple choice question.	A B C D
Predicting the next vehicle to go past the corner of my road	Car Truck Bus Lorry Bicycle Van Other
Tossing a coin	Heads Tails

Another word for experiment is *trial* and you should know is event. This often describes several outcomes put together. However, the term *event* is also often used to describe a single outcome.

B. ESTIMATING PROBABILITY

- Experimental estimation of probability

In many situations probabilities are estimated on the basis of data collected experimentally

Estimated $P(U) = \frac{n(U)}{n(T)}$

$P(U)$ = Probability the next outcome.
$n(U)$ = Number of Outcome.
$n(T)$ = Number of total event.

- Theoritical estimation of probability

There are, however, some situation where you do not need to collect data to make an estimate of probability. For example, when tossing a coin.

$P(H) = \frac{1}{2}$

$P(H)$ = Probability of the next toss showing heads.
1 = Number of ways getting the outcome heads.
2 = Total number of possible outcomes.

Express formally, the probability, P(A), of event A occuring is:

$P(A) = \frac{n(A)}{n(S)}$

P(A) = porbability of event A occuring.
N(A) = number of ways that event *A* can occur.
N(S) = Total number of ways that the possible events can occur

- Probabilities 0 and 1

The two extremes of probability are *certainty* at one end of the scale and impossibility at the other.

Experiment	Certain Event	Impossible Event
Rolling a single die	The result is in the range 1 to 6 inclusive	The result is a 7
Tossing a coin	Getting either heads or tails	Getting neither heads nor tails.

- Certainty

Probability of an event which is certain is one.

$\frac{n(A)}{n(s)} = 1$

n(A) = number of ways that the event can occur.
n(s) = total number of possible event.

- Impossibility

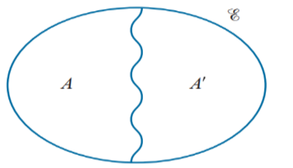
For impossible events, the number of ways that event can occur n(A), is zero.

$\frac{n(A)}{n(S)} = \frac{0}{n(S)} = 0$

- The complementof an event

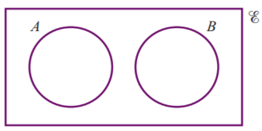
The complement of an event *A*, denoted by *A'*, is the event not-*A*, that is the event 'A does not happen'.

$P(A) + P(A') = 1$



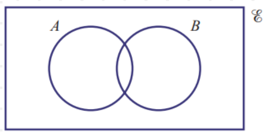
C. THE PROBABILITY OF EITHER ONE EVENTOR ANOTHER

Where two events, A and B, are mutually exclusive, the probability that either A or B occurs is equal to the sum of the separate probabilities of A and B occuring.



$P(A\ or\ B) = P(A) + P(B)$
$P(A \cup B) = P(A) + P(B)$

Where two events, *A* and *B*, are *not* mutually exclusive, the probability that either *A* or *B* occurs is equal to the sum of the separate probabilities of *A* and *B* occurring minus the probability of *A* and *B* occurring together.



$P(AorB) = P(A) + P(B) - P(A\ and\ B)$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

INDEPENDENT AND DEPENDENT EVENTS

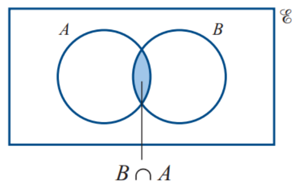
For two independent events, *A* and *B*,

$P(A \cap B) = P(A) \times P(B).$

In situation like this the possible outcomes resulting from the different experiments are often shown on a *tree diagram.*

CONDITIONAL PROBABILITY

Conditional probability is used when you're your estimate of the probability of an event is altered by your knowledge of whether some other event has occurred



$P(B|A) = \dfrac{P(B \cap A)}{P(A)}$

$P(B|A)$ = The probability of B given A

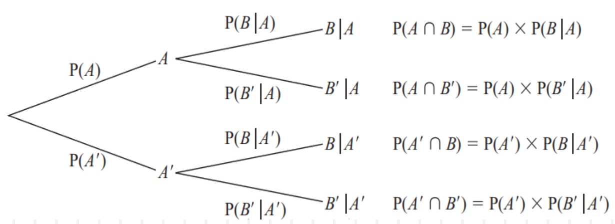
$P(B \cap A)$= The probability of both A and B.

$P(A)$ = The probability of A

If *A* and *B* are independent, then $P(B|A) = P(B|A')$ and this just *P(B).*
If *A* and *B* are dependent, then $P(B|A) \neq P(B|A')$

For dependent events $P(A \cap B) = P(A) \times P(B|A)$
For independent events $P(A \cap B) = P(A) \times P(B)$
$P(A \cap B)$ = the probability of both A and B occuring.
$P(B|A)$ = The probability of B occuring, given that A has occurred.
$P(A)$ = The probability of A occuring.

The ideas in the last example can be expressed more generally for any two dependent events, A and B in the tree diagram.



The tree diagram shows you that

- $P(B) = P(A \cap B) + P(A' \cap B)$
  $= P(A) \times P(B|A) + P(A') \times P(B|A')$
- $P(A \cap B) = P(A) \times P(B|A)$
  $P(B|A) = \dfrac{P(A \cap B)}{P(A)}$

Sample Question :

[9709_s14_qp_63_006]
1. Tom and Ben play a game repeatedly. The probability that tom wins any game is 0.3. Each game is wonby either Tom or Ben. Tom and Ben stop playing when one of them (to be called the champion) has won two games.

(i) Find the probability that Ben becomes the champion after playing exactly 2 games.
(ii) Find the probability that Ben becomes the champion.
(iii) Given that Tom becomes the champion, find the probability that he won the 2nd game.

[9709_s15_qp_61_003]
2. Jason throws two fair dice, each faces numbered 1 to 6. Event A 'one of the numbers obtained is divisible by 3 and the other number is not divisible by 3'. Event B 'the product of the two numbers obtained is even'.

(i) Determine whether events A and B are independent, showing your working
(ii) Are events A and B mutually excusive? Justify your answer.

[9709_s15_qp_63_004]
3. A pet shop has 9 rabbits for sale, 6 of which are white. A random sample of two rabbits is chosen without replacement.

(i) Show that the probability that exactly one of the two rabbits in the sample is white is $\frac{1}{2}$.
(ii) Cnstruct the probability distribution table for the number of white rabbits in the sample.
(iii) Find the expected value of the number of white rabbits in the sample.